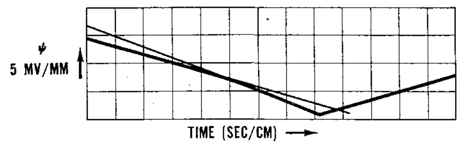

GYRO CONSTANTS

$H = 5\times10^5$ gram-cm^2/sec
$D_0 = 59.0\times10^3$ gram-cm^2/sec (measured)
$K_T = 75$ dyne-cm/ma
$K_P = 260$ mv/deg

Fig. 4 Actual plot of gyro pickoff signal vs time

float to rotate in the opposite direction. With the use of Eq. (14), where $\Delta\dot{\psi}$ either is measured directly or is an average quantity, input axis damping now can be computed.

Figure 4 is an actual trace of a gyro pickoff taken during input axis damping tests on one of Kearfott's experimental gyros. At the time of this test, the gyro was mounted in a polar configuration with its fixed restraint torques biased out by a precision supply. To obtain the trace, a current of 1 ma was fed into the gyro torquer. The resulting rates $\dot{\psi}_1$ and $\dot{\psi}_2$ were taken from the curve as 0.055 and 0.073 deg/sec, respectively.

With the constants given in Fig. 4 and with the use of Eq. (14), input axis damping was measured as 1.3×10^7 g-cm^2/sec. This figure provided an experimentally derived damping ratio of $D_i/D_0 = 220$.

Impossibility of Linearizing a Hot-Wire Anemometer for Measurements in Turbulent Flows

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Turbulence investigations are having to deal increasingly with situations in which the fluctuations are not much smaller than the mean velocity. It is well known that, in these circumstances, the nonlinearity of a hot-wire anemometer gives rise to large errors. This had led to the use of systems in which the electronic circuit contains some element with a reciprocal nonlinearity so that the output is linear with the fluid velocity. The authors wish to point out that there is a basic difficulty in this procedure. (Doubtless many workers are aware of this, but, to the knowledge of the authors, it never has been stated explicitly, and one gets the impression that sometimes it is overlooked. References 1 and 3 contain some related remarks.)

The difficulty arises from the fact that a hot wire responds to the instantaneous velocity V normal to its axis,‡ and the relationship between this and a single velocity component ($V = [(U + u_1)^2 + u_2^2]^{1/2}$ in the case of a single wire normal to the mean stream) is a nonlinear one. Electronic correction may be devised for nonlinearities between V and the signal obtained, e.g., King's law, but it is inherently impossible to correct for this further effect as the correction depends on u_2, which is not being measured independently. When one attempts to measure it by using X wires, the u_3 component comes in in the the same way to introduce large errors when there are large fluctuations.

The errors arising from this effect are of the same order as those involved in nonlinear, e.g., constant-current, operation in the following sense. When one analyzes linear operation by the series-expansion method (along the lines of Ref. 2, pp. 97–100), terms of the same type and degree appear. However, the errors may be substantially different because of differences in the coefficients of these terms. This depends on the particular quantity that is being measured, and it is difficult to generalize about the merits of linearization. Furthermore, the relevance of conclusions, which are reached on the basis of the first few terms of series expansions, to the flows with large fluctuations for which the matter is particularly important, is open to question. However, errors so estimated at least may illustrate the situation. A few remarks on the measurement of different quantities will be presented below. The notation is as follows: U is the mean stream speed; u_1, u_2, and u_3 are the components of the velocity fluctuation, the first being taken in the mean stream direction; and $< >$ signifies a mean value.

Hinze[2] has used the example of isotropic, normally distributed, normally correlated turbulence with $<u_1^2>^{1/2}/U = 0.2$ for estimating the seriousness of errors in hot wire anemometry. It will be convenient to follow suit here. However, some of the terms in the expansions, in general, are not assessed readily, and it is not clear how typical this example is. This is usually particularly true of the terms, such as the first two terms of Eq. (2-44) of Ref. 2, that involve fluctuating velocities to one degree higher than the quantity being measured. (These will be referred to below as first-order terms.) These terms are zero in isotropic turbulence, and then the error is produced by the second-order terms, but they are not zero in general. Next, measurement of particular quantities is considered.

1. Mean Velocity Measured with a Single Wire

The expression given by a linear relationship between velocity and signal analogous to Eq. (2-40) of Ref. 2 is

$$U_{\text{meas}} = U_{\text{act}}[1 + (\tfrac{1}{2}<u_2^2>/U^2) + \ldots]$$

Hence, linearized operation is no better, and probably worse, than the straightforward constant-current operation for U measurement. This is the most drastic of the present conclusions. One would not, of course, construct linearizers just for mean-velocity measurements, but, having constructed them for turbulence measurements, one also might use them for measuring mean velocities and suppose that there was less error than usual. Perhaps it is not appreciated always that this supposition is fallacious.

2. $<u_1^2>$ Measured with a Single Wire

The expression for linearized operation corresponding to Eqs. (2-46) and (2-47) of Ref. 2 is

$$<u_1^2>_{\text{meas}} = <u_1^2>_{\text{act}}\left[1 + \frac{<u_1u_2^2>}{U<u_1^2>} + \frac{1}{4}\frac{<u_2^4>}{U^2<u_1^2>} - \frac{1}{4}\frac{<u_2^2>}{U^2<u_1^2>} - \frac{<u_1^2u_2^2>}{U^2<u_1^2>} + \ldots\right]$$

For isotropic, normally distributed, and normally correlated turbulence with 20% intensity, this indicates a 2% error, and linearization is markedly advantageous. Moreover, apart

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‡ The further complication of the effect of yaw is being ignored. There are circumstances in which this is not allowable,[1] but one can try to avoid those circumstances, whereas the point being discussed here always will arise.

from a small doubt about the first-order term, this advantage is likely to be maintained in more general circumstances.

3. $<u_2^2>$ Measured with an X Wire

Similar considerations for this case lead to

$$<u_2^2>_{meas} = <u_2^2>_{act}[1 - 2(<u_2^2u_3^2>/U^2<u_2^2>) + \ldots]$$

and so indicate an 8% error for linearized operation in the particular circumstances just specified. Although there is still likely to be an improvement in most circumstances over, say, constant-current operation, note that this error is of the order-of-magnitude more usually associated with nonlinearized systems.

4. Reynolds Stress Measured with an X Wire

This case is of particular interest, because the first-order terms are of the same form as the turbulent-transport terms in the energy-balance equation of a turbulent shear flow, and existing experimental information may be used to assess them. For linearized operation

$$<u_1u_2>_{meas} = <u_1u_2>_{act}[1 + (<u_2u_3^2>/U<u_1u_2>) - 2(<u_1u_2u_3^2>/U^2<u_1u_2>) + \ldots]$$

If, as in two-dimensional flow, $<u_1\, u_2\, u_3^2> = <u_1\, u_2>\cdot<u_3^2>$, the second-order term gives 8% error with 20% intensity. Experimental data on turbulent shear flows suggest that a typical value of $|<u_2u_3^2>/<u_1u_2><u_1^2>^{1/2}|$ is 0.5, and, in this case, the first-order term gives 10% error for 20% intensity. (The two terms may be of the same or opposite sign according to the circumstances.) This suggests that, for the greater intensities at which these points are most important, second-order terms probably will be more serious than first-order terms. By way of comparison, constant-current operation gives

$$<u_1u_2>_{meas} = <u_1u_2>_{act}\left[1 - \frac{3}{4}(1+2\alpha)\frac{<u_1^2u_2>}{U<u_1u_2>} - \frac{1}{4}(1+2\alpha)\frac{<u_2^3>}{U<u_1u_2>} + \frac{<u_2u_3^2>}{U<u_1u_2>} + \frac{5+12\alpha+12\alpha^2}{8}\left(\frac{<u_1^3u_2> + <u_1u_2^3>}{U^2<u_1u_2>}\right) - \frac{7+6\alpha}{2}\frac{<u_1u_2u_3^2>}{U^2<u_1u_2>} - \frac{1+12\alpha+20\alpha^2}{8}\times\left(\frac{<u_1^2> + <u_2^2>}{U^2}\right) + \frac{1+6\alpha}{2}\frac{<u_3^2>}{U^2} + \ldots\right]$$

where α is the same as in Eq. (2-45) of Ref. 2. The first-order terms usually will give larger errors than in linearized operation. The second-order terms are too complicated for any general assessment, but for the particular case of $<u_1^2> = <u_2^2> = <u_3^2>$, $<u_1u_2u_3^2> = <u_1u_2>\cdot<u_3^2>$, and $<u_1^3\, u_2> = <u_1\, u_2^3> = (8/\pi)^{1/2}<u_1u_2><u_1^2>$ (chosen because $<|u^3|> = (8/\pi)^{1/2}<u^2>^{3/2}$ for a Gaussian distribution), these terms give a markedly smaller error than the terms in a linearized operation. This is a very restricted inference, but in general there is perhaps little to choose between the two modes of operation for $<u_1u_2>$ measurements.

As already implied, these considerations are illustrative, not definitive. But the authors think that they lead to the following (interrelated) inferences: that the advantages of linearization are sufficiently marginal that it may not always be worth the trouble and expense; that the weight given to measurements with linearized equipment as opposed to nonlinearized ought not to be so much greater; and that the accuracy of results from linearized equipment needs to be assessed in terms of the particular quantity and circumstances. In connection with this last point, it may be considered an advantage of linearization that the estimation of errors is relatively straightforward, not merely because the algebra is less laborious but also because it involves fewer quantities whose magnitude has to be guessed.

Of course, the pros and cons of different methods cannot be considered in terms of accuracy alone. Some people express a preference for linearized operation because it is more straightforward to use. However, one gets the impression that measurements with linearized equipment sometimes have been given greater weight, although accuracy may not have been the reason for selecting this technique. The main point is that this calls for caution.

Since preparation of this note, a paper by Rose[4] has appeared dealing with the same matter, though from a different point of view.

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Deceleration and Its Higher Time Derivatives for Objects During Atmospheric Entry

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Expressions are derived for the first and second time derivatives of deceleration of an object during atmospheric entry. The altitudes at which the maximum and minimum values of these derivatives occur are determined. The differences between these altitudes and the altitude at which maximum deceleration occurs are shown to be constants that depend only on the atmospheric density. The maximum and minimum values of the first and second time derivatives of deceleration are determined. It is shown that these values are independent of the drag characteristics of the object.

Nomenclature

V = velocity of entering object along trajectory (positive in direction of motion)
t = time
C_D = drag coefficient
ρ_0 = atmospheric density at earth's surface
A = reference area for drag evaluation
V_E = initial velocity of entering object
β = atmospheric density coefficient such that $\rho = \rho_0 e^{-\beta y}$
m = mass of object
θ_E = angle of entry (measured with respect to local horizon)

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